Water management in Ontario

Ontario
Water Resources
Commission

Great Lakes
Water Quality
Surveys Program

LAKESHORE TWO-DIMENSIONAL

DISPERSION

Great Lakes Surveys
Ontario Water Resources Commission
135 St. Clair Ave. W.,
Toronto 195, Ontario

M. D. Palmer,
Environmental Engineer

J. B. Izatt,
Scientific Applications Programmer

March 26, 1970

LAKESHORE TWO-DIMENSIONAL DISPERSION

ABSTRACT

Hourly two-dimensional dispersion characteristics are determined from recording current meter histories for the nearshore areas on lakes Erie and Ontario. The current histories were obtained in areas within 4 km of shore and at water depths of 10 to 14 metres during May to November 1968. A Markov chain process was applied to hourly current readings. Three different formulations of the stochastic process were tested prior to the selection of the most reliable one. The results obtained in applying the developed technique compare favourably with results obtained from conventional dye injection and drogue studies.

KEY WORDS: currents, diffusion, instruments and techniques, nearshore.

GREAT LAKES NEARSHORE
TWO-DIMENSIONAL
HOURLY DISPERSION

TABLE OF CONTENTS

LIST OF TABLES

GREAT LAKES NEARSHORE
TWO-DIMENSIONAL
HOURLY DISPERSION

LIST OF FIGURES

LAKESHORE TWO-DIMENSIONAL DISPERSION

INTRODUCTION

Measurements of the nearshore currents in the Great Lakes have revealed the highly variable nature of the water movements in both time and space. As the currents are the basic mechanism responsible for dispersion, it is necessary to understand and develop analytical techniques to describe the water movements. The problem can be simplified by considering different time and spatial scales separately. This technique has been used in air pollution studies (Hilst (4)), estuarine analysis (Okubo (7)) and Great Lakes Studies (Palmer (9)). In the Great Lakes studies, monthly average dispersion and five hour dispersion characteristics were developed from the continuous history of currents measured at a fixed point every 10 minutes and averaged to hourly values. The five hour diffusion characteristics were determined after lengthy analysis on the computer for the four major compass directions. However, the computations are seriously limited to time periods not greater than five hours and major compass directions by computer time and core space considerations. This represents a serious limitation if the technique is to be incorporated as an integral step into a larger assimilation model. The purpose of this paper is to outline alternate methods for determining hourly two-dimensional dispersion character-

istics from current meter records for depths of water 10 to 14 metres within 4 km of the shore. These methods require a fraction of the computer time compared to the other method (Palmer (9)). The methods also produce maximum, mean and minimum dispersion characteristics with the associated probabilities for eight major compass directions and various time periods.

DEVELOPMENT OF METHOD

From the continuous hourly current records for a month, it is possible to obtain a one-hour state probability transition matrix. The currents are first classified into 80 designated current states (8 directions and 10 speed intervals) as outlined in Table 1.

TABLE 1. State number index

	Direction				
Magnitude	337.5° to 22.5°	22.5° to 67.5°		247.5° to 292.5°	292.5° to 337.5°
0 - 2 cm/sec.	1	9	-------	7	8
2 - 4 cm/sec.	9	10	-------	15	16
-					
-					
-					
16 - 18 cm/sec.	65	66	-------	71	72
18 - 20 cm/sec.	73	74	-------	79	80

The velocity intervals can be varied to suit the measured velocity ranges. An initial state probability vector "q" for the month can be obtained. The initial state probability vector "q" is the probability of occurrence of each state for the month. A transitional state probability matrix can be developed. As an example, suppose a loaded die was thrown 1,000 times and a record was kept of the results. There are six possible outcomes, called states, on each throw; namely one, two, three --- or six. Suppose a two is cast, it is possible to determine the probability of all the other possible states occurring on the next throw by examining the record to obtain what states followed a two state and how often it occurred. A different set of probabilities would be obtained by considering a different start or initital state; namely one, or three, Considering all possibilities produces a transitional state probability matrix six by six. The one-hour state probability transition matrix can then be obtained from the analysis of the current histories classified into states (Palmer (9)). A sample of the resulting 80 x 80 transition matrix appears in Table 2A.

TABLE 2A. One hour transition probability matrix

Final State

	SPEED		0 to 4 cm/sec.		2 to 4 cm/sec.		
		ANGLE	337.5° to 22.5°	292.5° to 337.5°	337.5° to 22.5°	292.5° to 337.5°	- - -
Initial State	0 to 2 cm/sec.	337.5-22.5 - - - - - - 292.5-337.5			A		
	2 to 4 cm/sec.	337.5-22.5 - - - - - - 292.5-337.5					
					etc.		

Each block represents an 8 x 8 matrix and a sample representing block "A"may be found in Table 2B.

TABLE 2B. One hour transition probability matrix

		Final State 2.0 to 4.0 cm/sec.			
		337.5° to 22.5°	22.5° to 67.5°	---	292.5° to 337.5°
Initial State 0 to 2.0 cm/sec.	337.5° to 22.5°	0.01737	0.00668	---	0.00240
	22.5° to 67.5°	0.00634	0.01692	---	0.00121
	- - -	- - -	- - -	--- --- ---	- - -
	292.5° to 337.5°	0.00626	0.00521	---	0.01147

Hence by selecting an initial current a row is specified and the probability of any state one hour later may be found by looking at the intersection of that row and the appropriate column. For example, the first element of block "A" represents the probability of going from a state of 0-2 cm/sec. and 337.5° to 22.5° to a state of 2-4 cm/sec. and 337.5° to 22.5°.

Three alternate methods were developed using the above techniques to predict dispersion characteristics. As a first step, it was necessary to develop the state probability vector after a certain number of hours given the initial state vector and the continuous

current meter records. The three approaches involve different methods of determining this final state vector but use the same method of predicting the dispersion characteristics.

The first method involved combining the initial state probability vector "q_m" for the month (essentially a frequency analysis of a month's current meter record by states) with various powers of the one hour transition probability matrix "$T^i{}_{mn}(1)$" where i represents the power of the matrix (Bharucha-Reid (1); Kemeny (5)).

$$q_n(k) = \sum_{m=1}^{80} q_m \, T^i{}_{mn}(1) \quad \ldots\ldots\ldots\ldots\ldots (1)$$

where q_n = final state probability vector after "k" hours. "n" is the final state.

q_m = initial state probability vector. "m" is the initial state.

$T^i{}_{mn}(1)$ = one hour state transition probability matrix raised to "i^{th}" power where i = k.

The obvious weakness of this method is that any errors present in the first power of the transition matrix will be propagated and intensified by the process of raising the matrix to power "i".

The second laternative method involves the use of several first order transition matrices based on varying the intervals between successive currents used in computing the matrix. For example, a matrix

could be found by considering only the probability of transitions involving every fifth hourly reading and hence a first order matrix for five hours could be found. These matrices could then be combined with the initial state vector to give the final state vector after any given number of hours.

$$q_n(k) \quad = \quad \sum_{m=1}^{80} q_m \, T_{mn}(k) \quad \ldots\ldots\ldots\ldots\ldots (2)$$

where $q_n(k)$ = final state probability vector after "k" hours. "n" is the final state.

q_m = initial state probability vector. "m" is the initial state.

$T_{mn}(k)$ = state probability transition matrix after k = 1, 2 , 3 --- hours.

This method avoids the necessity of raising the transition matrix to higher powers and thus eliminates errors found in the previous method. However, computer time increases substantially when it becomes necessary to search the monthly records many times to establish the various first order matrices. The method is weakened somewhat in theory because a first order Markov chain implies that a result is dependent only on the directly preceding result. While it is known that currents occurring in a particular hour are related to currents occurring one hour previously, the relationship of currents separated in time by five or eight hours is dependent on the physical size of the

area as well as the driving force. It is not possible to state as a general rule that the current occurring now is dependent on the current which occurred five hours earlier.

The final method tested avoided the weaknesses of the previous two methods. The initial state vector and the transition matrix are defined as in the first method. However, the higher order final states are computed using the final state vector for the previous hour as the initial state vector for the present computation. For instance, the final state vector for the first hour would be multiplied by the first order one-hour transition matrix to give the final state vector for the second hour (Gablinger (3)).

$$q_n(k) \quad = \quad \sum_{m=1}^{80} q_m(k-1) \; T_{mn}(1) \ldots\ldots\ldots (3)$$

where $q_n(k)$ = final state probability vector after "k" hours

$q_m(k-1)$ = final state probability vector after "k-1" hours

$T_{mn}(1)$ = one hour state probability transition matrix.

This method directly relates the results of one time period to subsequent time periods.

The question of how many hours must be considered for short term dispersion characteristics can be resolved by determining the

70-11-GL

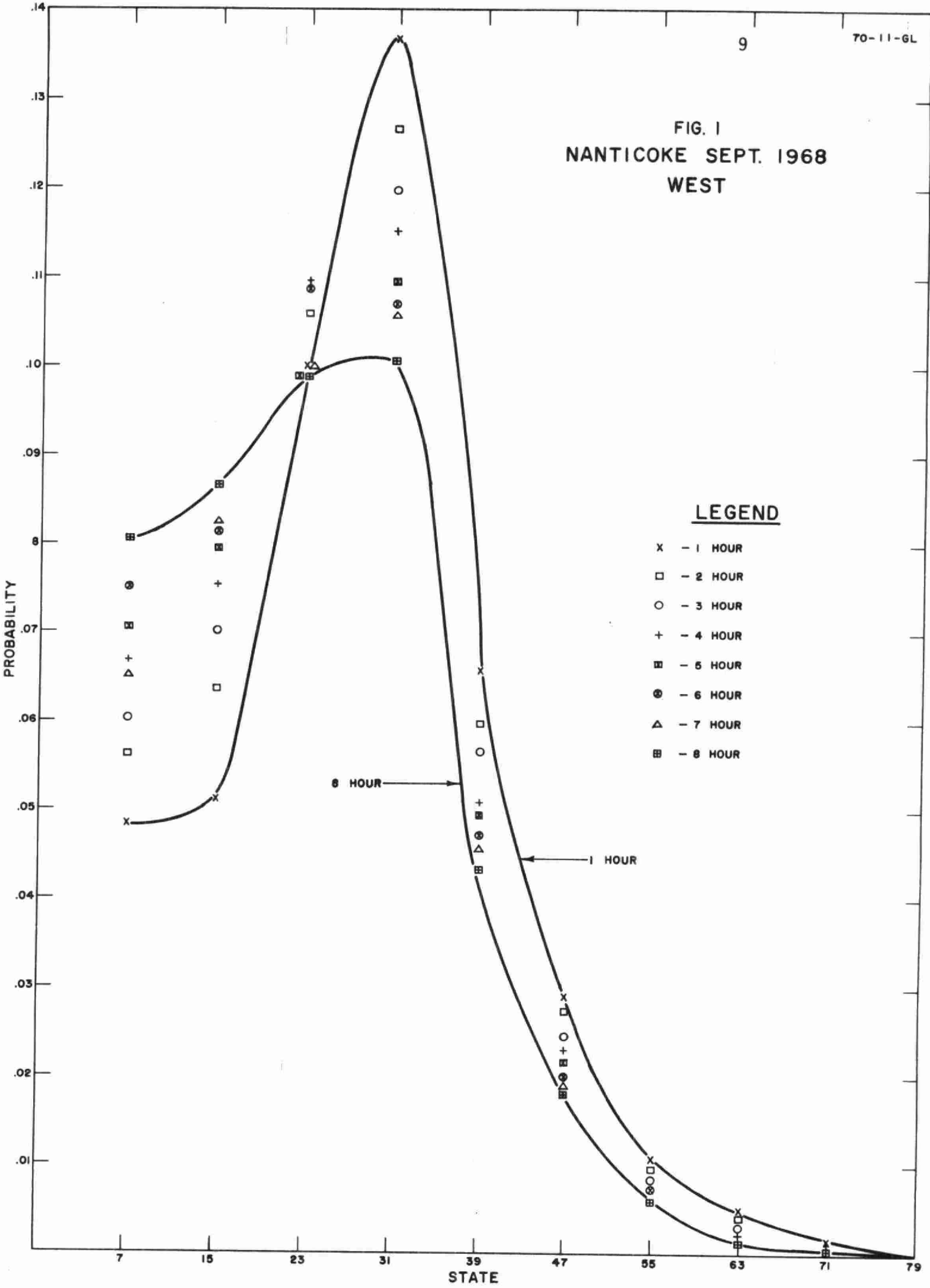
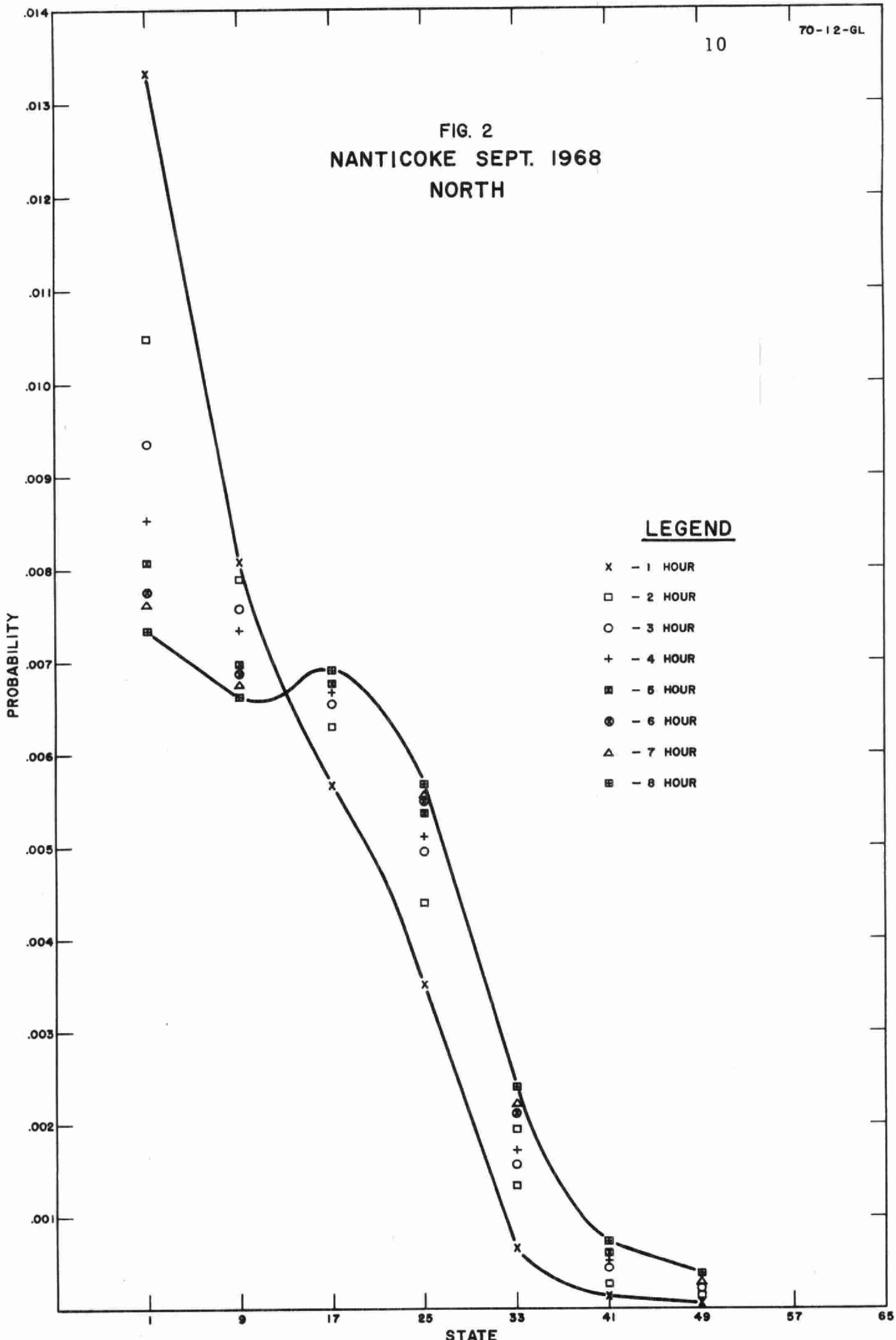


FIG. 1
NANTICOKE SEPT. 1968
WEST


70-12-GL
FIG. 2
NANTICOKE SEPT. 1968
NORTH
LEGEND
X - 1 HOUR
□ - 2 HOUR
○ - 3 HOUR
+ - 4 HOUR
- 5 HOUR
- 6 HOUR
△ - 7 HOUR
- 8 HOUR
PROBABILITY
.014
.013
.012
.011
.010
.009
.008
.007
.006
.005
.004
.003
.002
.001
STATE
1
9
17
25
33
41
49
57
65

number of hours required to approach a monthly mean. In Figures 1 and 2, the state probabilities are plotted against the various states for west and north respectively. The monthly mean state for the west direction obtained from a frequency analysis of the monthly record is state 23. Figure 1 demonstrates the tendency of the most probable state to approach state 23 by the eighth hour, whereas a most probable state for the north direction is state 1 which is achieved in the first and successive hours. As the various directions at a particular location are related, it is necessary to consider the longest time interval required to approach mean conditions in any direction. It has been found that a period of eight hours is a judicious time interval for short term dispersion characteristics.

A measure of the dispersion characteristics for each hourly interval can be obtained by using the state probability for that time interval as weighting factors. The weighting factors multiplied by the respective states and extended to the time interval considered will generate the most probable distances travelled in the eight major compass directions for the time period considered. As an example, the distance travelled in the south direction after one hour is computed as follows:

State	Mean Velocity cm/sec.	Probability	
(A)	(B)	(C)	(B) x (C)
1	1	0.0132	0.0132
9	3	0.0081	0.0162
17	5	0.0057	0.0285
25	7	0.0034	0.0238
33	9	0.0009	0.0081
41	11	0.0002	0.0021
49	13	0.0001	0.0013
		Sum =	0.0932

Most probable distance travelled after one hour $= 0.0932 \times 60 \times 60$
$= 336$ cm.

A comparison of the results by using equations 1, 2 and 3 for the mean eight hour pattern appears in Figure 3. It is observed that there is little difference in employing the three methods although the third method was selected as being the more reliable when considering the propagation of errors. A plot of the successive hourly patterns produced by the third method appear in Figures 4 and 5 for a location on Lake Erie and Lake Ontario respectively. The areas defined by the various related hourly contours are representative of the dispersion characteristics for that time period. The patterns DO NOT represent actual dispersion plumes as the probabilities have been combined

FIG. 3

FRENCHMAN BAY

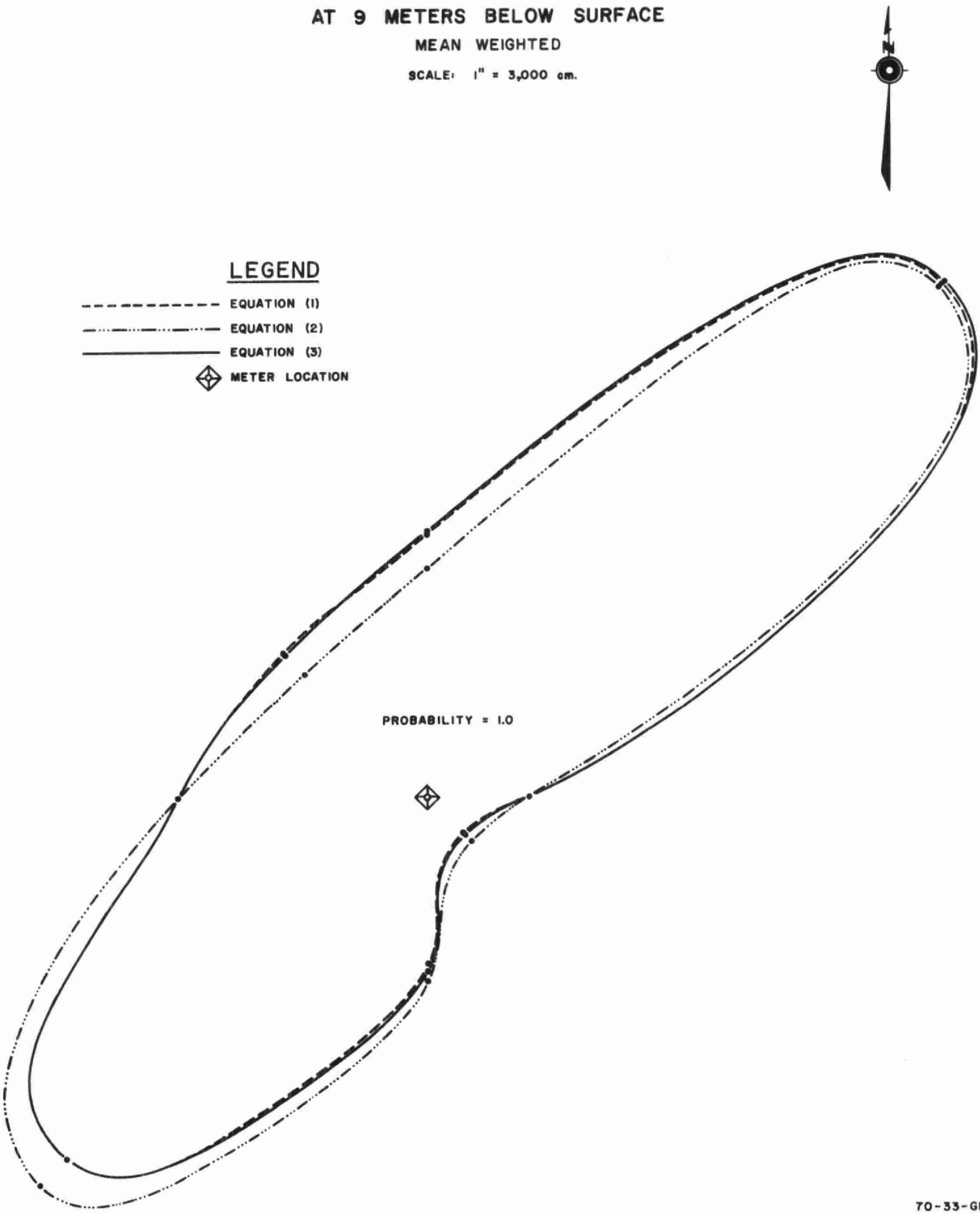
JUNE 1968

EIGHT HOUR DISPERSION

AT 9 METERS BELOW SURFACE

MEAN WEIGHTED

SCALE: 1" = 3,000 cm.



70-33-GL

FIG. 4

NANTICOKE

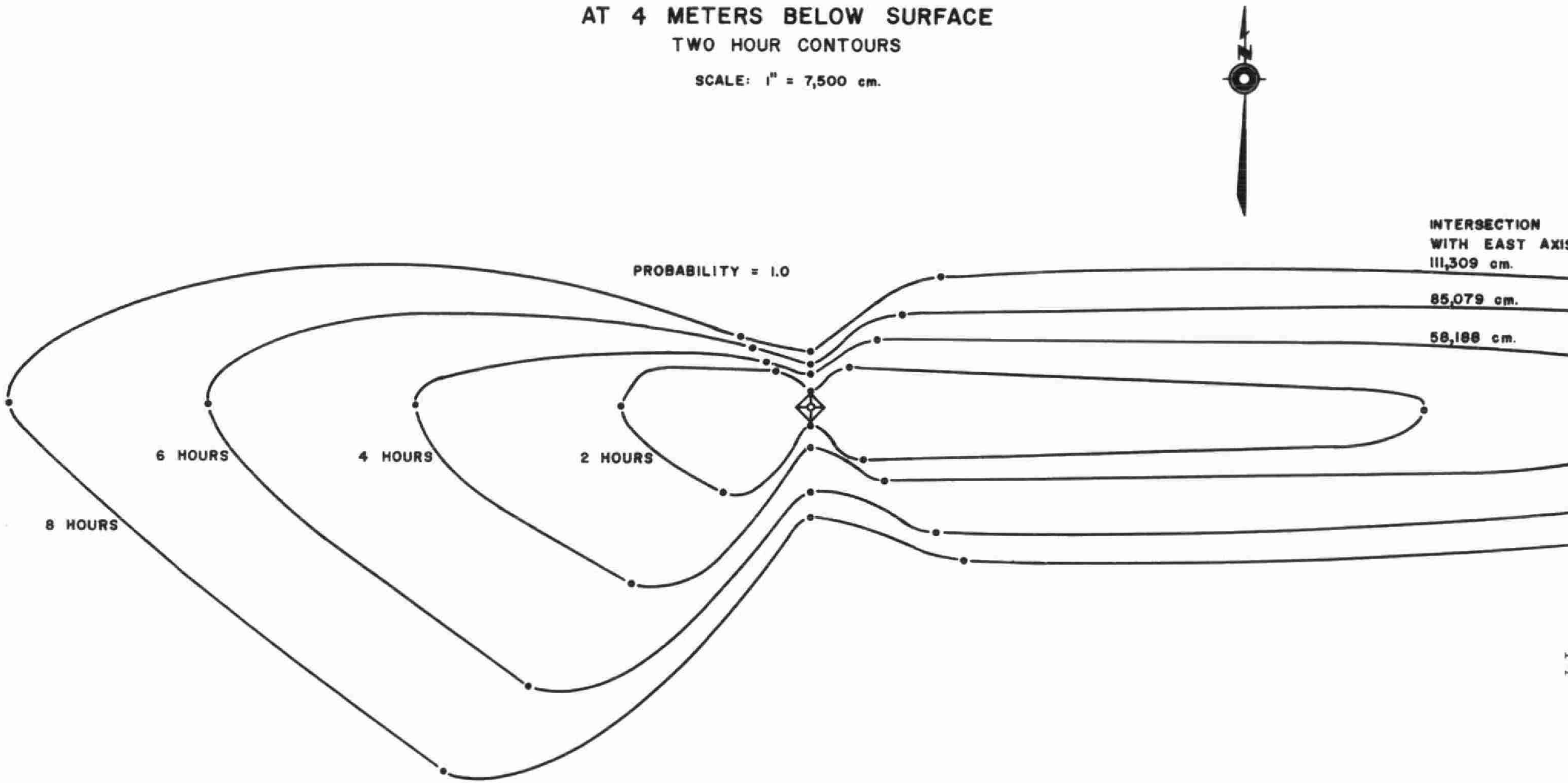
SEPT. 1968

MEAN HOURLY DISPERSION

AT 4 METERS BELOW SURFACE

TWO HOUR CONTOURS

SCALE: 1" = 7,500 cm.



70-13-GL

FIG. 5

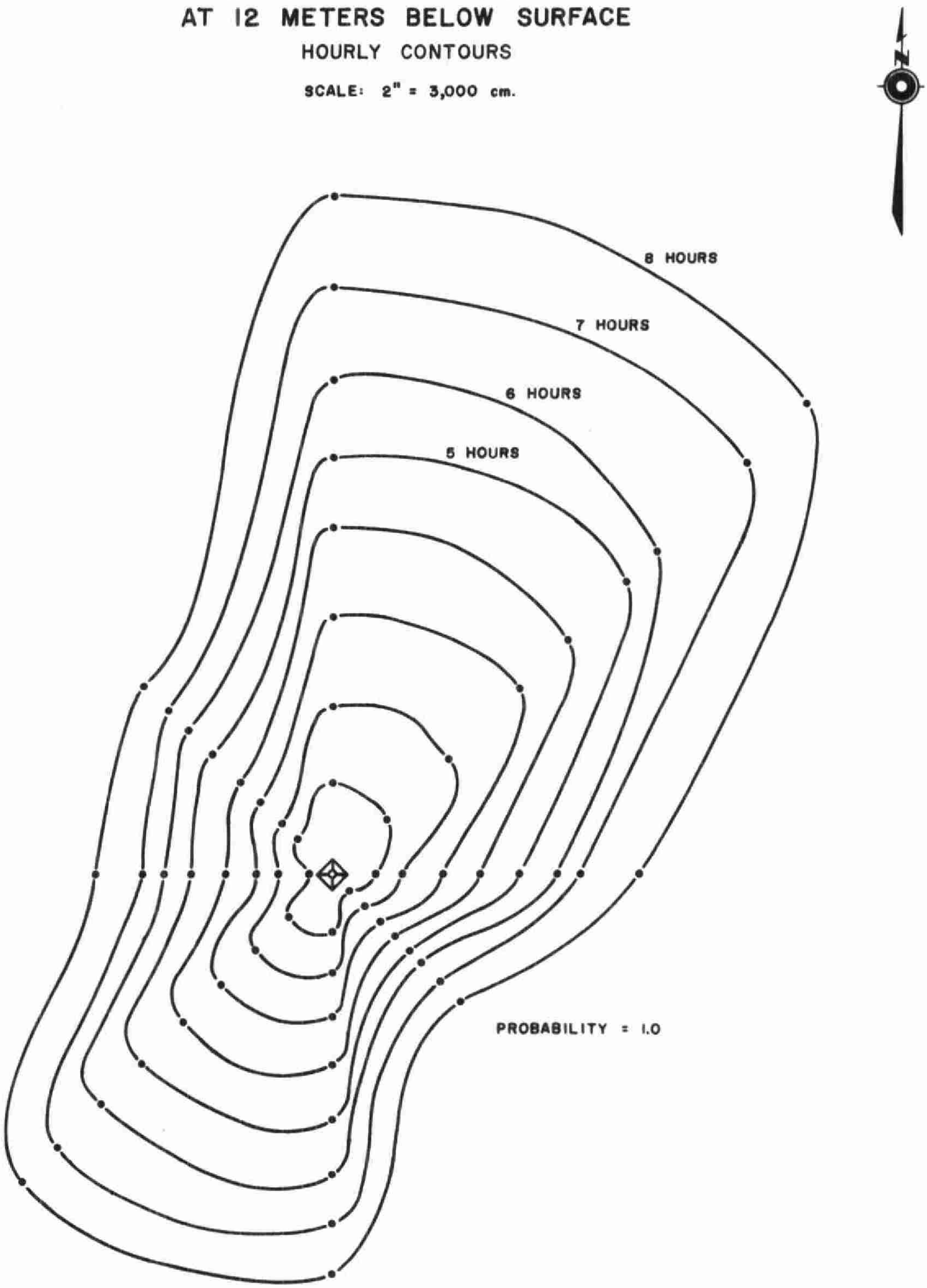
FRENCHMAN BAY

JUNE 1968

MEAN HOURLY DISPERSION
AT 12 METERS BELOW SURFACE

HOURLY CONTOURS

SCALE: 2" = 3,000 cm.



70-14-GL

with distances travelled in any time period. A two-dimensional coefficient "ϵ" can be obtained by dividing the area contained by the related contours by the time period considered. A sample of the diffusion coefficients computed by this method appears in Table 3.

TABLE 3. Diffusion coefficients for Nanticoke September 1968

Time Hrs.	ϵ Diffusion Coefficients $cm^2/sec.$
1	0.67×10^4
2	1.6×10^4
3	3.7×10^4
4	5.7×10^4
5	8.3×10^4
6	11.1×10^4
7	13.9×10^4
8	17.4×10^4

It is observed that the diffusion coefficients increase with time. This is an expected trend as the longer time periods permit the larger scale physical phenomena to become operative and increase the dispersion characteristics, e.g. lake wide wind seiches and littoral drift effects become operative at periods of greater than four hours on the average. A comparison of the diffusion coefficients obtained by this method with results from other studies appear in Table 4.

TABLE 4. Results of other studies

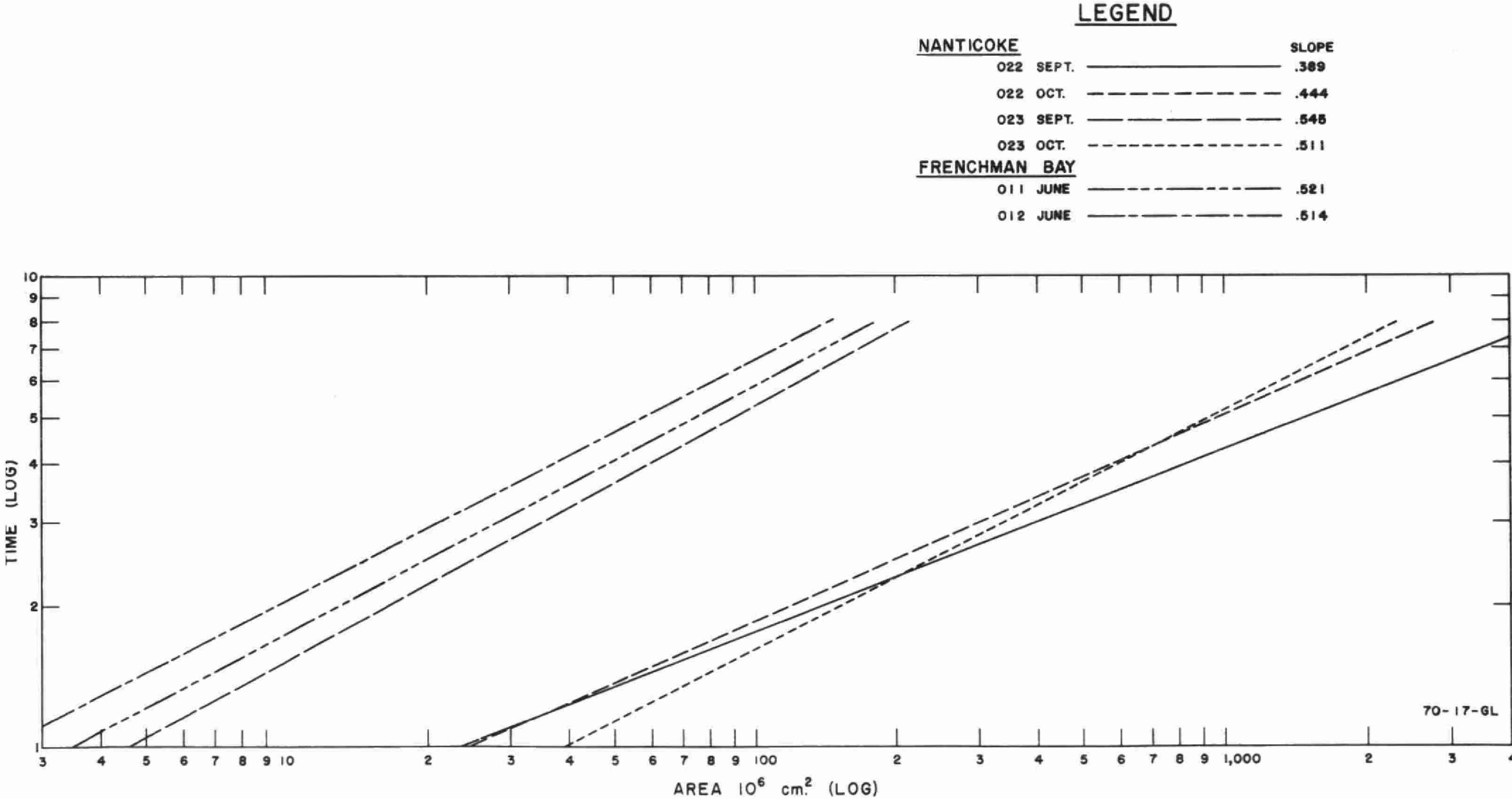
Reference		(cm^2/sec.)
Csanady	(2)	4×10^2 (one at 2×10^3)
Okubo	(8)	3 to 6×10^4
Noble	(6)	2.44×10^2

The mean conditions for four different locations on lakes Erie and Ontario appear in Figure 6. It is observed that a power relationship exists between the log (time) and the log (area). This is a similar situation determined by Okubo (7) in his estuarine study where the diffusion coefficient increases with distance from an estuary mouth, if one assumes that distance and time are directly related. The power varies from .389 to .545 which indicates the dependence of diffusion on both the local topography and the time of year.

It is also possible to obtain a measure of the minimum and maximum likely dispersion characteristics from the meter records. This can be achieved by considering various portions of the state probability vector at the end of various time intervals. For instance, a measure of the minimum dispersion characteristics could be obtained by considering only states 1 to 8. The probabilities for these states can be summed and weighting factors can be obtained by increasing the probabilities to sum to one. The distances travelled for each time

FIG. 6

MEAN DISPERSION AREAS



interval can then be obtained by multiplying the weighting factors by the state velocity intervals and the appropriate time intervals considered. The probability of this condition is defined by the sum of the original probabilities for these states. Similarly, a maximum condition can be determined by considering only states above state 41 and performing a similar operation to determine the weighting factors. A selection of the states to be considered for minimum and maximum condition will obviously be dependent upon the current history for a particular area. A comparison of the results for equations 1, 2 and 3 for the minimum and maximum eight hour patterns appears in Figures 7 and 8. For the minimum there is little difference between the three methods, however, equation 2 does generate a significantly different pattern for the maximum condition. There are some theoretical problems in selecting method two (equation(2)) and the third method (equation (3)) was selected to determine minimum and maximum as well as the mean patterns. The results of applying equations (1), (2) and (3) to the data to determine minimum, mean and maximum eight hour dispersion patterns appear in Figures 7, 3 and 8. The areas defined by the different methods appear in Table 5.

FIG 7
FRENCHMAN BAY
JUNE 1968
EIGHT HOUR DISPERSION
AT 9 METERS BELOW SURFACE
MINIMUM WEIGHTED
SCALE: 1" = 2,000 cm.

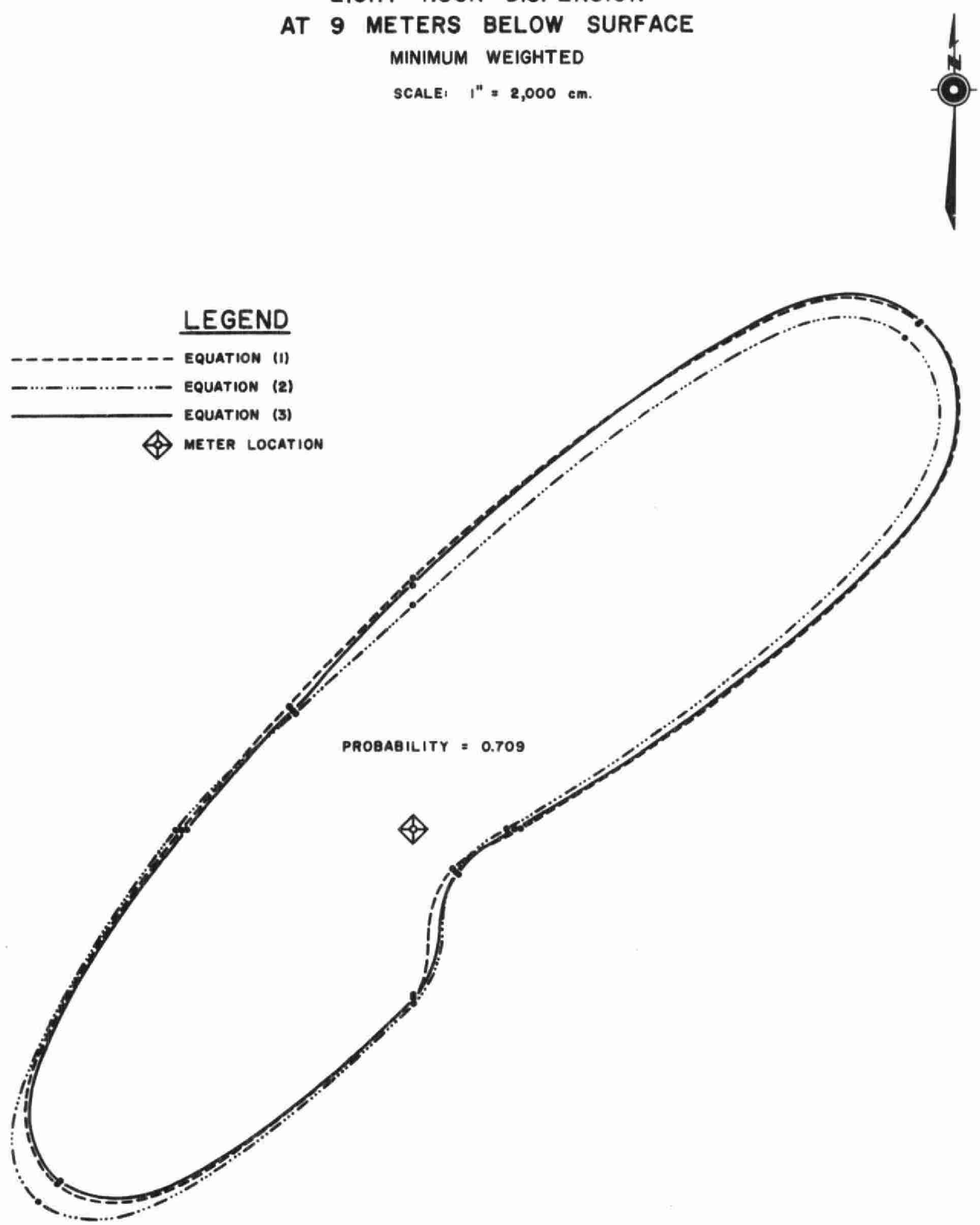


FIG. 8

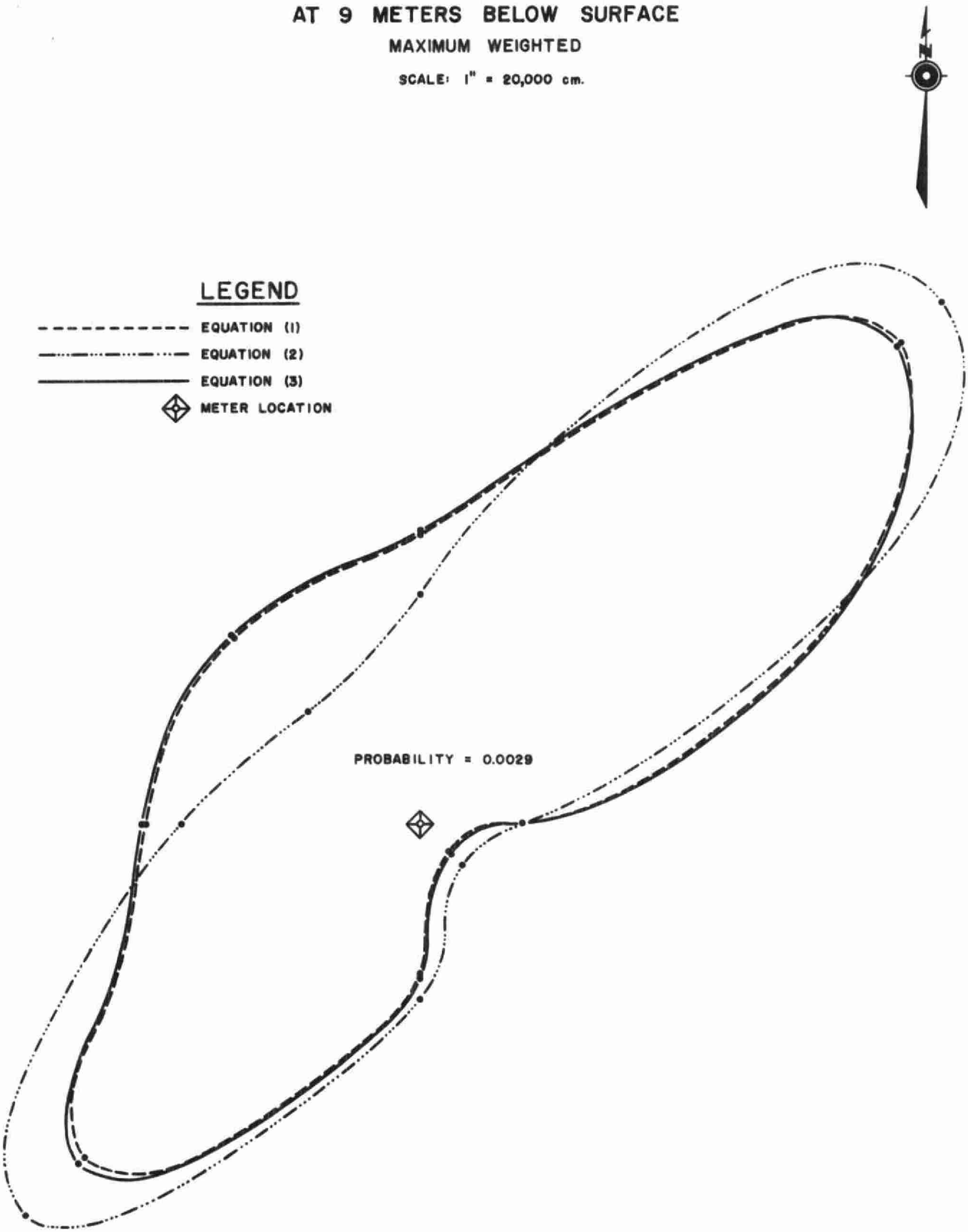
FRENCHMAN BAY

JUNE 1968

EIGHT HOUR DISPERSION
AT 9 METERS BELOW SURFACE

MAXIMUM WEIGHTED

SCALE: 1" = 20,000 cm.



70-34-GL

FIG. 9

NANTICOKE

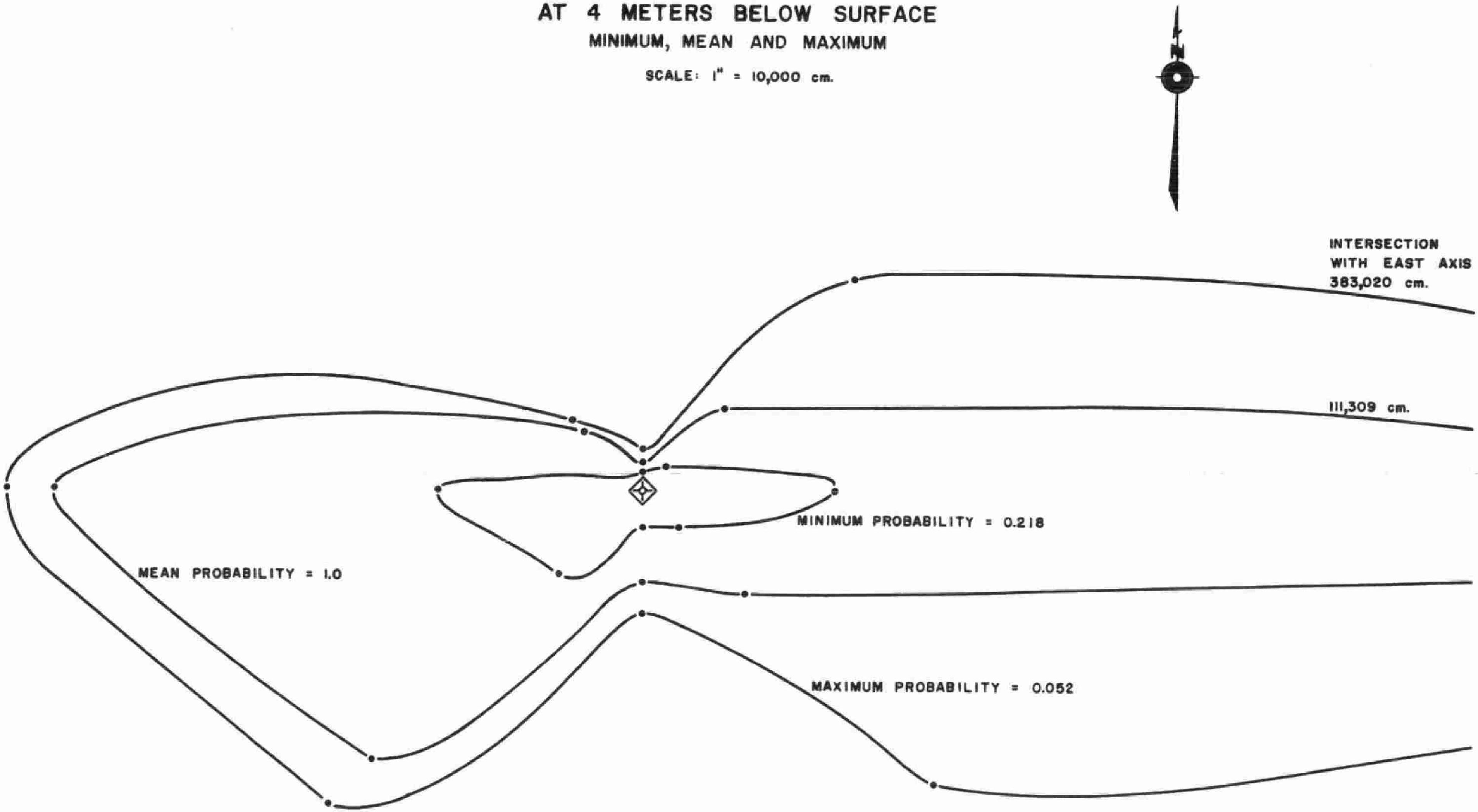
SEPT. 1968

EIGHT HOUR DISPERSION

AT 4 METERS BELOW SURFACE

MINIMUM, MEAN AND MAXIMUM

SCALE: 1" = 10,000 cm.



70-15-GL

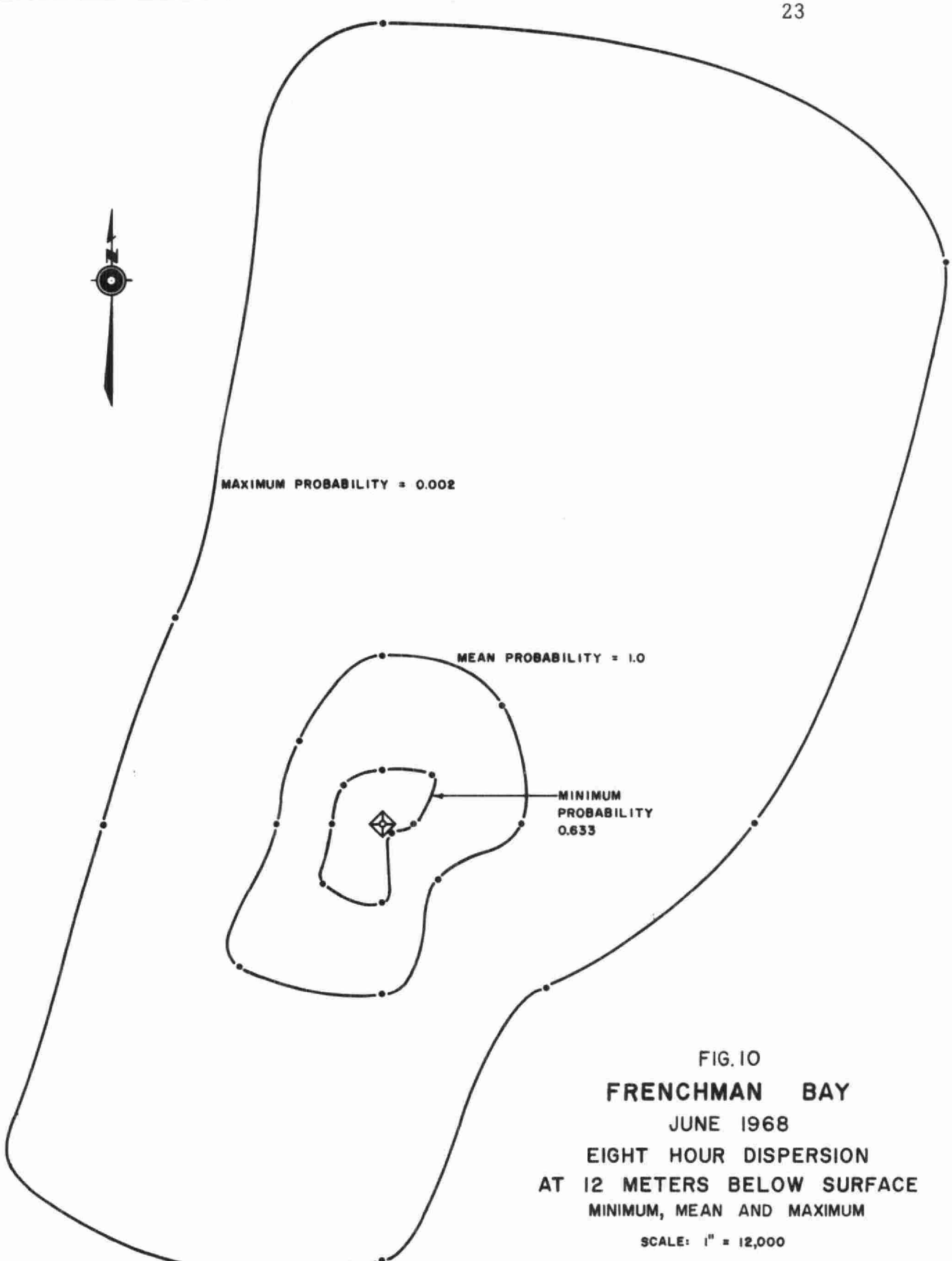


FIG. 10
FRENCHMAN BAY
JUNE 1968
EIGHT HOUR DISPERSION
AT 12 METERS BELOW SURFACE
MINIMUM, MEAN AND MAXIMUM
SCALE: 1" = 12,000

70-16-GL

FIG. 11

MINIMUM AND MAXIMUM DISPERSION AREAS

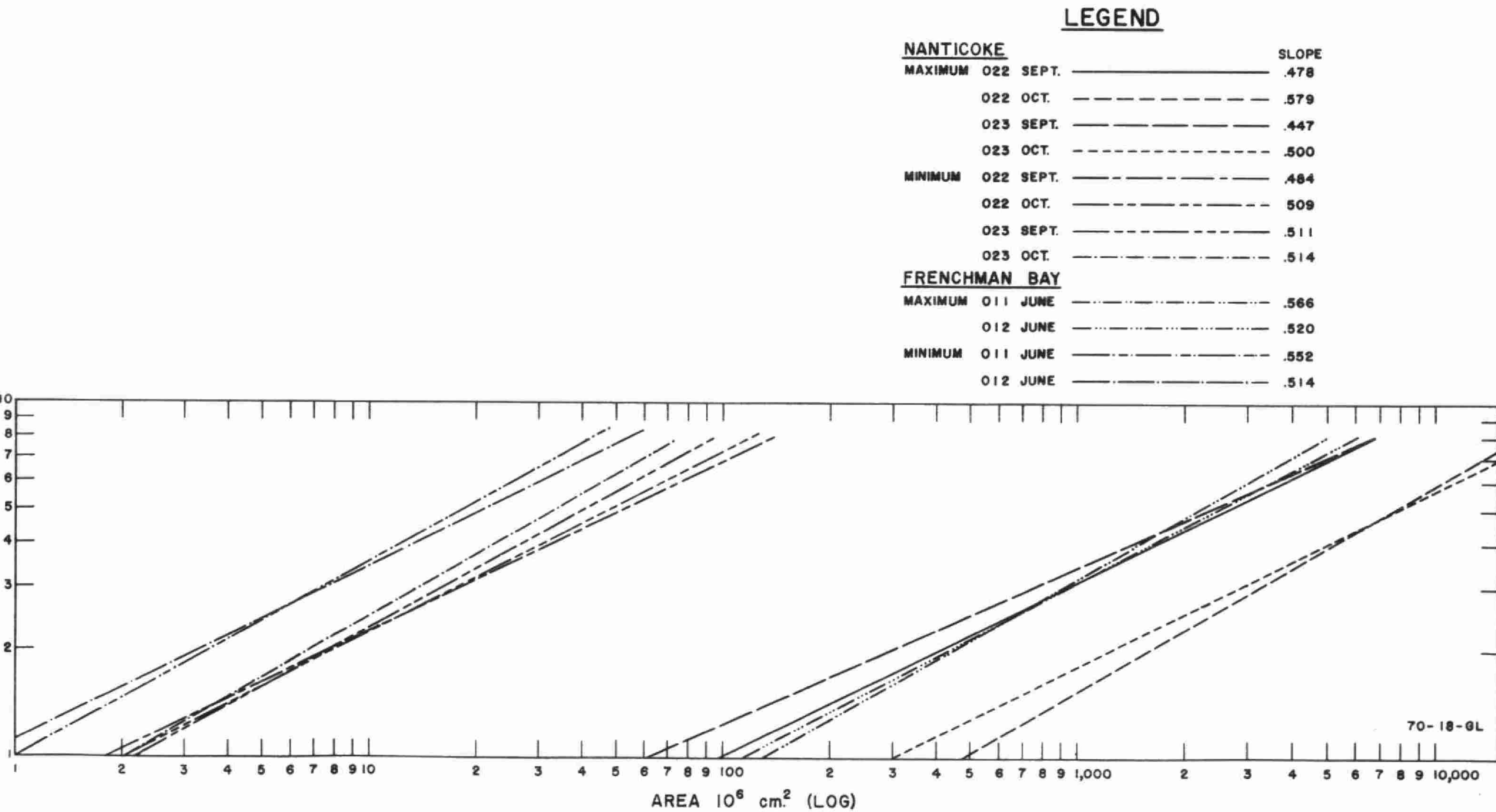


TABLE 5. Eight hour area comparisons

	Method		
	Equation (1) $\times 10^7$ cm^2	Equation (2) $\times 10^7$ cm^2	Equation (3) $\times 10^7$ cm^2
Minimum	5.2	4.5	4.7
Mean	14.0	10.2	14.4
Maximum	6,600	5,200	6,000

Examples of the maximum and minimum conditions found using the third method mentioned above for an eight hour period appear in Figures 9 and 10. Once again, the area is defined by the various hourly contours which are representative of the dispersion characteristics. The areas defined by this method appear in Figure 11 for the maximum and minimum power at four different locations on lakes Erie and Ontario. It will be noticed that a power relationship exists once again for a maximum and minimum condition, but that the slopes do not vary as widely as those of the mean, although the intercepts are more variable.

SUMMARY AND CONCLUSIONS

The Markov chain analytical technique has been demonstrated as a reasonable method for determining two-dimensional dispersion characteristics from recording current meter data. The figures of

patterns DO NOT represent actual plumes. It has been tested in three different forms, namely, the development of higher order chains utilizing the power of first order chains, higher order chains developed for different time intervals, and step wise, first order Markov chains applied hour after hour. The latter technique has been found to be the most reliable to predict hourly two-dimension hourly dispersion patterns However, the difference between the three methods is, in most cases, negligible. The results obtained by employing the Markov process compare favourably with dispersion characteristics determined by conventional techniques, such as dye dispersion runs and drogue trackings. Further, the Markov chain process can be particularly useful in determining the limits of short term (hourly) dispersion characteristics.

A linear relationship exists between the log (time) plotted against the log (area) with slopes varying from 0.389 to 0.579 and intercepts varying 0.026 to 1.1 hours. This is a similar result determined for estuarine dispersion. No significant relationship could be developed relating diffusion characteristics with month or geographic location from this data. The highly variable nature of the nearshore area dispersion characteristics of lakes was confirmed.

REFERENCES

1. BHARUCHA-REID, A.T., Elements of the Theory of Markov Processes and Their Applications, McGraw-Hill Book Co. Inc., 1960, pp. 10-13.

2. CSANADY, G.T., 1964. Hydrodynamic Studies on Lake Huron at Baie du Dore, Summer 1964. Water Resources Institute, University of Waterloo, PR 19, 1965.

3. GABLINGER, M. and D. P. LOUCKS, 1970, Markov Models for Flow Regulation. A. S. C. E. Jour. of Hydraulics Div., v. 96, HYI, Jan. p.165.

4. HILST, G.R., 1968. An Air Pollution Model of Connecticut Proc. IBM Scientific Symposium Water and Air Resource Management, p.251.

5. KEMENY, J.G. and J.L. SNELL, Finite Markov Chains. D. Van Nostrand Co. Inc., 1962, pp. 14-34.

6. NOBLE, V.E., 1961. Measurements of Horizontal Diffusion in the Great Lakes, Proc 4th Conf. on Research in the Great Lakes, p.85.

7. OKUBO, A., 1964. Equations Describing the Diffusion of an Introduced Pollutant in a One-Dimensional Estuary. Studies on Oceanography. Ed. Koji Yoshida, Univ. of Washington Press, p. 216.

8. OKUBO, A. and J. S. FARLOW, 1967. Analysis of Some Great Lakes Drogue Studies. Proc 10th Conf. on Great Lakes Research. p.299.

9. PALMER, M.D. and J.B. IZATT, 1969. Great Lakes Nearshore Modelling from Current Meter Data. Ontario Water Resources Report, 135 St. Clair Ave. W., Toronto.

NOTATION

The following symbols are used in this paper:

	=	two-dimensional dispersion coefficient $cm^2/sec.$
$q_n(k)$	=	final state probability vector after "k" hours
q_m	=	initital state probability vector
$T^i{}_{mn}(k)$	=	state transition probability matrix for "k" hour intervals between readings raised to the power "i".
current	-	a horizontal (two-dimensional) movement of water which has magnitude (cm/sec.) and direction (degrees)
diffusion coefficient	-	a measure of the mean spread of particles transported by water movements or a measure of dilution ($cm^2/sec.$)
initial state probability vector	-	the probability of occurrence of all possible states for a month arranged as a column or row vector
littoral drift	-	the water movement generated from the interaction of waves and shorelines
Markov chain	-	an array of probabilities which relate the state existing now with the state which will occur in the next time period. It is a method of describing a sequence of related events.
spatial	-	a term relating physical or geometrical separation
state	-	physical possible condition that a variable can assume

		e.g. $1^{\circ}C$ is a temperature state 2 cm/sec. at 170° is a current state 300 to 320 umhos/cm is a conductivity state
state transition probability matrix	-	similar to Markov chain but restricted to a two-dimensional array
wind seiche	-	a periodic water movement resulting from the interaction of winds and water surfaces in areas with finite dimensions